GENERALIZED ROE'S NUMERICAL SCHEME FOR A TWO-FLUID MODEL

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ABSTRACT

This paper is devoted to a mathematical and numerical study of a six equation two-fluid model. We will prove that the model is strictly hyperbolic due to the inclusion of the virtual mass force term in the phasic momentum equations. The two-fluid model is naturally written under a nonconservative form. To solve the nonlinear Riemann problem for this nonconservative hyperbolic system, a generalized Roe's approximate Riemann solver, is used, based on a linearization of the nonconservative terms. A Godunov type numerical scheme is built, using this approximate Riemann solver.

0. Introduction

The model considered here, is a first order equal pressure two-fluid model. Except the virtual mass force term, which contains partial derivatives, the other terms of mass and momentum transfer between phases are assumed to be absent. They will appear as source terms, and will be added to the numerical scheme. The resulting model is a nonconservative hyperbolic one.

In this paper, we present a numerical method based upon an approximate Riemann solver. Such numerical schemes have been widely used for hyperbolic systems of conservation laws. These schemes were originally developed for Gas dynamics calculations, and have been extended to conservative two-phase flow model [1]. The purpose of this paper is to extend this Riemann solver approach to a nonconservative hyperbolic two-fluid model.

1. Two-Phase Flow Model

1.1 Equations

We consider the first order differential equations of mass, momentum and energy conservation of the two-fluid model which might describe two-component two-phase flow in a straight pipe.

Phasic mass balance

$$\partial_t(\alpha_k\rho_k) + \partial_x(\alpha_k\rho_k u_k) = 0 \qquad \text{(I.1)}$$

Phasic momentum balance

$$\partial_t(\alpha_k\rho_k u_k) + \partial_x(\alpha_k\rho_k u_k^2) + \alpha_k\partial_x p = M_{vm}^k \qquad \text{(I.2)}$$

Phasic energy balance

$$\partial_t(\alpha_k\rho_k H_k) - \alpha_k\partial_t p + \partial_x(\alpha_k\rho_k u_k H_k) = M^k_{vm}u_k \tag{I.3}$$

with

$$H_k = h_k + \frac{u_k^2}{2} \tag{I.4}$$

and

$$\alpha_v + \alpha_l = 1 \tag{I.5}$$

Here the subscript k=l refers to the liquid phase and k=v to the vapor phase; ρ_k, u_k, h_k and α_k are the mass density, the velocity, the enthalpy and the void fraction of the k-phase. P is the pressure assumed to be equal in the two phases and ρ is the mixture density given by

$$\rho = \rho_v\alpha_v + \rho_l\alpha_l$$

To close the system the liquid phase is assumed to be incompressible with constant mass density ρ_l while the vapor mass density is given by the following state equation :

$$\rho_v = \rho_v(p, h_v)$$

The results, however, can be generalized to a compressible liquid phase.

We have chosen the following formulation for the virtual mass force term M_{vm} at the right hand side of (I.2) and (I.3) :

$$M_{vm} = -\alpha_v\alpha_l c_{vm}(\partial_t(u_v - u_l) + u_l\partial_x u_v - u_v\partial_x u_l)$$

where c_{vm} is the coefficient of virtual mass. Such formulation is used in the RELAP5 code [2], and is derived from the following expression suggested by Drew et al. [3]

$$M_{vm} = -\alpha_v\rho_l c_{vm}\{\partial_t(u_v - u_l) + u_v\partial_x(u_v - u_l) + (u_v - u_l)\,((\lambda - 2)\,\partial_x u_v + (1-\lambda)\,\partial_x u_l)\}$$

where λ is a void fraction dependent parameter. For many cases of interest, the inclusion or neglect of virtual mass force in the phasic momentum equations does not appreciably change the momentum results. However, the inclusion of this term with its temporal and spatial derivative terms changes the hyperbolicity of the system. In general [4], the computation efficiency of the numerical scheme is improved.

The virtual mass force term that we have chosen, is not the only possible one. Another formulation will give similar results. The value of the virtual mass coefficient c_{vm} will be defined so as to have a hyperbolic system (see theorem I.1).

System (I.1)-(I.3) is not written in a conservative form. We introduce the mixture momentum ρu , and the mixture total energy ρE, given by

$$\rho u = \rho_v\alpha_v u_v + \rho_l\alpha_l u_l,$$

$$\rho E = \rho_v\alpha_v E_v + \rho_l\alpha_l E_l$$

with $E_k = e_k + \frac{1}{2}u_k^2$ and $e_k = h_k - \frac{p}{\rho_k}$. These mixture quantities satisfy the following conservation equations

$$\partial_t\rho u + \partial_x(\alpha_v\rho_v u_v^2 + \alpha_l\rho_l u_l^2) + \partial_x p = 0$$

$$\partial_t\rho E + \partial_x(\alpha_v\rho_v u_v H_v + \alpha_l\rho_l u_l H_l) = 0$$

The nonconservative terms in (I.2)-(I.3) arise from the splitting of this latter equations into two separate phase momentum and energy equations.

Introducing the mixture quantities, $\rho, \rho u, \rho E$, and the concentration variable c defined by

$$c = \frac{\alpha_v \rho_v}{\alpha_v \rho_v + \alpha_l \rho_l}$$

we can write, for smooth solutions, system (I.1)-(I.3) in the following nonconservative form

$$M_\delta(\mathbf{u})\,\partial_t \mathbf{u} + A_\delta(\mathbf{u})\,\partial_x \mathbf{u} = 0 \tag{I.6}$$

with

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} \rho c \\ \rho(1-c) \\ \rho u \\ \rho(1-c)\,u_l \\ \rho E \\ \rho(1-c)\,E_l \end{bmatrix}$$

$$M_\delta(\mathbf{u}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \delta(u_3-u_4)\,u_2/u_1^2 & -\delta u_4/u_2 & -\delta u_2/u_1 & 1+\delta+\delta u_2/u_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_\delta(\mathbf{u}) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ p_1 - u_v^2 & p_2 - u_l^2 & p_3 + 2u_v \\ \alpha_l p_1 + \delta u_v u_4/u_1 & \alpha_l p_2 - \delta u_v u_l - u_l^2 & \alpha_l p_3 - \delta u_4/u_1 \\ \bar{u}p_1 - u_v H_v & \bar{u}p_2 - u_l H_l + (u_l - u_v)\,p/\rho_l & \bar{u}p_3 + H_v \\ \alpha_l u_l p_1 & \alpha_l u_l p_2 - u_l E_l & \alpha_l u_l p_3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ p_4 + 2(u_l - u_v) & p_5 & p_6 \\ 2u_l + \alpha_l p_4 + \delta u_3/u_1 & \alpha_l p_5 & \alpha_l p_6 \\ \bar{u}p_4 + H_l - H_v & \bar{u}p_5 + u_v & \bar{u}p_6 + u_l - u_v \\ \alpha_l u_l p_4 + E_l & \alpha_l u_l p_5 & \alpha_l u_l p_6 + u_l \end{bmatrix}$$

where $\delta = \frac{\alpha\rho}{\rho_l} c_{vm}$ is a parameter depending on the virtual mass coefficient and $\bar{u}$ is a velocity defined by

$$\bar{u} = \alpha_v u_v + \alpha_l u_l$$

The pressure derivatives p_i are given by

$$p_i = \left(\frac{\partial p}{\partial u_i}\right)_{u_j \neq u_i}$$

In the sequel, we will use the following systems of variables, depending on the computation to be performed

$$\mathbf{u} = (\rho c, \rho(1-c), \rho u, \rho(1-c)u_l, \rho E, \rho(1-c)E_l)$$
$$\mathbf{v} = (\rho c, \rho(1-c), \rho u, u_l, \rho E, \rho(1-c)e_l) \tag{I.7}$$

We can shift from one system of variables to another through a C^1 diffeomorphism.

Neglecting in the system (I.6) the terms arising from the virtual mass force leads to the condensed form:

$$\partial_t \mathbf{u} + A_0(\mathbf{u})\partial_x \mathbf{u} = 0 \tag{I.8}$$

with $A_0(\mathbf{u}) = A_{\delta=0}(\mathbf{u})$. This system is still non conservative due, for instance, to the term $(1-\alpha)\partial_x p$ in the liquid phase momentum equation. However, we prove the proposition [5]:

Proposition I.1. *Let Ω be a set of physical states defined by*

$$\Omega = \{u \;/\; (\rho > 0),\ (c \in [0,1])\}$$

Let $\mathbf{u}$ *be a continuous solution of system (I.1)-(I.3) and* $\mathbf{v}$ *be the vector valued function defined by (I.7). The function* $\mathbf{v}$ *is a solution of the following conservative system :*

$$\partial_t v + \partial_x g_0(v) = 0 \tag{I.9}$$

with the flux function defined by

$$g_0(v) = \begin{bmatrix} \rho c u_v \\ \rho(1-c)u_l \\ \rho c u_v^2 + \rho(1-c)u_l^2 + p \\ \dfrac{u_l^2}{2} + \dfrac{p}{\rho_l} \\ \rho c u_v H_v + \rho(1-c)u_l H_l \\ \rho(1-c)u_l e_l \end{bmatrix} \tag{I.10}$$

Proposition I.1 shows that the systems (I.8) and (I.9) are equivalent for smooth solutions.

1.2 Hyperbolicity of the System

In order to study the hyperbolicity of the system (I.6) we are looking for the eigenvalues of this system. To determine these eigenvalues we must find the six roots of a polynomial of degree six. We prefer to assume the relative velocity between the two phases much lower than the speed of sound of the two-phase mixture c_m. This is the case in many physically interesting configurations, for example for steam and water. Then, we introduce the following small parameter

$$\xi = \frac{(u_v - u_l)}{c_m}$$

and we use a perturbation method around $\xi = 0$ in order to know the behavior of the eigenvalues $\lambda_i(\mathbf{u})$, $i = 1, 6$.

Theorem I.1 *Let c_{vm}^0 be defined by :*

$$c_{vm}^0 = (4c(1-c))^{1/2}$$

We can find a positive number ξ_0 so that for any u that lies in the set Ω^* defined by

$$\Omega^* = \{u \in \Omega:\ (|\xi| \le \xi_0),\ (c_{vm} \ge c_{vm}^0)\}$$

all the eigenvalues of the system (I.6) are real and distinct.

For the proof of this theorem we refer the reader to reference 5. Theorem (I.1) shows that the inclusion of virtual mass force term, with $c_{vm} \ge 1$, makes the model well-posed. The computations have been done using a constant virtual mass coefficient, equal to 2.0 .

2. Numerical Method

2.1 A Weak Formulation of Roe's Approximate Riemann Solver

To solve the nonlinear Riemann problem for hyperbolic systems of conservation laws

$$\begin{aligned} &\partial_t \mathbf{u} + \partial_x \mathbf{f}(\mathbf{u}) = 0 \\ &\mathbf{u}(x,0) = u_L \ \ (x<0),\ \ \mathbf{u}(x,0) = u_R\ (x>0) \end{aligned} \tag{II.1}$$

Roe [6] introduces a local linearization

$$\partial_t \mathbf{u} + A(u_L, u_R)\,\partial_x \mathbf{u} = 0 \tag{II.2}$$

where $A(u_L, u_R)$ is some average Jacobian matrix, known as a Roe averaged matrix, constructed to have the crucial property

$$\mathbf{f}(u_R) - \mathbf{f}(u_L) = A(u_L, u_R)\,(u_R - u_L) \tag{II.3}$$

Such a matrix was first constructed by Roe for Euler equations with perfect gases[6], and then several extensions to real gases have been proposed (see reference 7 and the references therein).

This method does not apply to the nonconservative hyperbolic system (I.6), since the matrix $A_\delta(\mathbf{u})$ is not the derivative of a flux function $f_\delta(\mathbf{u})$. To overcome this difficulty, we will use a weak formulation of Roe's approximate Riemann solver introduced in reference 7. This formulation is based on the defintion of nonconservative products proposed by Dal Maso[8]. Using this definition, the authors deduce the following generalized Rankine-Hugoniot condition [9]:

$$\int_0^1 \left(-\sigma M_\delta + A_\delta(\Phi(s,u_L,u_R))\right) \frac{\partial \Phi}{\partial s}(s,u_L,u_R)\,ds = 0 \tag{II.4}$$

where $\Phi(s,u_L,u_R)$ is a Lipshitz continuous path connecting u_L and u_R.

As these jump conditions depend on the path Φ, it is necessary to add some information to the first order system (I.6). Here, we consider approximate solutions to the Riemann problem for the system (I.6) which are exact solutions to the approximate linear problem :

$$\begin{aligned} &M_\delta(u_L, u_R)\,\partial_t \mathbf{u} + A(u_L, u_R)_\Phi\,\partial_x \mathbf{u} = 0 \\ &\mathbf{u}(x,0) = u_L \ \ (x<0),\ \ \mathbf{u}(x,0) = u_R\ (x>0) \end{aligned} \tag{II.5}$$

where $A_\delta(u_L, u_R)_\Phi$ and $M_\delta(u_L, u_R)$ are constant matrices depending on the data (u_L, u_R), on the path Φ and satisfying the following properties :

$$A_\delta(u_L, u_R)_\Phi\,(u_R - u_L) = \int_0^1 A_\delta(\Phi(s,u_L,u_R)) \frac{\partial \Phi}{\partial s}(s,u_L,u_R)\,ds \tag{II.6}$$

$$M_\delta(u_L, u_R)_\Phi\,(u_R - u_L) = \int_0^1 M_\delta(\Phi(s,u_L,u_R)) \frac{\partial \Phi}{\partial s}(s,u_L,u_R)\,ds \tag{II.7}$$

which shows that shocks of the linear system satisfy the generalized Rankine Hugoniot condition (II.4).

2.2 Conservative System Case

We remark that for a conservative system, the matrix M_δ is equal to the identity and A_δ is the jacobian matrix of a flux function $f_\delta(\mathbf{u})$. Then, the right hand side of (II.6) is independent of the path Φ :

$$\int_0^1 A_\delta(\Phi(s,u_L,u_R))\frac{\partial\Phi}{\partial s}(s,u_L,u_R)\,ds = f_\delta(u_R) - f_\delta(u_L)$$

Thus, (II.6) coincides exactly with Roe's condition (II.3). A shock wave solution of the linearized system satisfies the Rankine Hugoniot condition for the nonlinear conservative system (II.1), and is independent on Φ. In this case, the path Φ is only useful to linearize the Jacobian matrix $A_\delta(\mathbf{u})$ to obtain $A_\delta(u_L, u_R)_\Phi$.

To construct such matrix we follow the method introduced in reference 7. The main feature is the choice of the canonical path for a parameter vector $\mathbf{w}$:

$$\Phi(s,u_L,u_R) = \psi_0(w_L + s(w_R - w_L)) \tag{II.8}$$

where ψ_0 is a smooth function such that $\psi_0(w_L) = u_L$, $\psi_0(w_R) = u_R$ and $G_0(\mathbf{w}) = \partial\psi_0/\partial\mathbf{w}$ is a regular matrix for every state $\mathbf{w}$.

Using this path, we define Roe's matrix by [7]:

$$A_\delta(u_L, u_R)_\Phi = C(u_L, u_R)_\Phi B(u_L, u_R)_\Phi^{-1} \tag{II.9}$$

with

$$B(u_L, u_R)_\Phi = \int_0^1 G_0(w_L + s(w_R - w_L))\,ds \tag{II.10}$$

$$C(u_L, u_R)_\Phi = \int_0^1 A G_0(w_L + s(w_R - w_L))\,ds \tag{II.11}$$

The choice of the canonical path is motivated by the results obtained for the Euler equations (see reference 7 for more details on the effect of the path upon averaging).

2.3 Nonconservative System Case

To construct an approximate Riemann solver for hyperbolic nonconservative systems, we can use the above method since the flux function $f_\delta(\mathbf{u})$ does not appear explicitly in (II.9). However, for such systems, the choice of the path Φ will be important. Besides the linearization of the matrix $A_\delta(\mathbf{u})$, both the exact solution and the approximate solver strongly dependent to the path Φ.

We propose to separate the path contributions on the two above problems :

- the defintion of weak solutions which need a path with a physical meaning.
- the linearization of the nonlinear matrix which does not need a physical path.

In the next section we present a strategy for the construction of an approximate Riemann solver for a nonconservative hyperbolic system. First, we choose an average for the nonconservative terms so as to get a conservative system. This step amounts to the choice of a physical path for the nonconservative products. Then, we construct an approximate Riemann solver for the resulting conservative system as described in the above section.

3. Application to the Two-Fluid Model

In this section, using the above weak formulation, we build a Roe's approximate Riemann solver for the two-fluid model (I.1)-(I.3). First, we construct the Roe-averaged matrix for the basic system (I.8) which does not include the virtual mass force term. Then, we will extend the method for the complete system.

3.1 Two-Fluid Model without Virtual Mass Force Term

Proposition II.1 *Let* $\bar{\alpha}_l = 1 - \bar{\alpha}$ *be defined by*

$$\frac{1}{1-\bar{\alpha}} = \frac{1}{2}\left(\frac{1}{1-\alpha_R} + \frac{1}{1-\alpha_L}\right) \tag{III.1}$$

Let u *be a weak solution of the conservative system*

$$\partial_t u + \partial_x h(u, \bar{\alpha}) = 0 \tag{III.2}$$

with the following flux function

$$h(u, \bar{\alpha}) = \begin{bmatrix} \rho c u_v \\ \rho(1-c)u_l \\ \rho c u_v^2 + \rho(1-c)u_l^2 + p \\ \rho(1-c)u_l^2 + (1-\bar{\alpha})p \\ \rho c u_v H_v + \rho(1-c)u_l H_l \\ \rho(1-c)u_l\left(H_l - \dfrac{\bar{p}}{\rho_l}\right) \end{bmatrix} \tag{III.3}$$

The vector valued function $v = \varphi(u)$ *given by (I.7) is a weak solution (smooth or shock solution) of the conservative system (I.10)*

For the proof of this proposition see reference 7. System (III.2) is a conservative one. Thus, the path Φ will have an effect only upon the linearization of the nonlinear Jacobian matrix

$$A(u, \bar{\alpha}) = \frac{\partial h(u, \bar{\alpha})}{\partial u} .$$

We apply the method presented in the above section with the parameter vector chosen as follows

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} \sqrt{\rho c} \\ \sqrt{\rho(1-c)} \\ \sqrt{\rho c}\,u_v \\ \sqrt{\rho(1-c)}\,u_l \\ \sqrt{\rho c}\,H_v \\ \sqrt{\rho(1-c)}\,H_l \end{bmatrix} \tag{III.4}$$

and $\psi_0(w)$ given by the following expression

$$\psi_0(w) = \begin{bmatrix} w_1^2 \\ w_2^2 \\ w_1 w_3 + w_2 w_4 \\ w_2 w_4 \\ w_1 w_5 + w_2 w_6 - p \\ w_2 w_6 - (1-\bar{\alpha})p \end{bmatrix} \tag{III.5}$$

Straightforward computations yield

$$B(u_L, u_R)_\Phi = \begin{bmatrix} 2\bar{w}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\bar{w}_2 & 0 & 0 & 0 & 0 \\ \bar{w}_3 & \bar{w}_4 & \bar{w}_1 & \bar{w}_2 & 0 & 0 \\ 0 & \bar{w}_4 & 0 & \bar{w}_2 & 0 & 0 \\ \bar{w}_5 - \tilde{p}_{w1} & \bar{w}_6 - \tilde{p}_{w2} & \tilde{p}_{w3} & \tilde{p}_{w4} & \bar{w}_1 - \tilde{p}_{w5} & \bar{w}_2 - \tilde{p}_{w6} \\ -\bar{\alpha}_1\tilde{p}_{w1} & \bar{w}_6 - \bar{\alpha}_1\tilde{p}_{w2} & -\bar{\alpha}_1\tilde{p}_{w3} & -\bar{\alpha}_1\tilde{p}_{w4} & -\bar{\alpha}_1\tilde{p}_{w5} & \bar{w}_2 - \bar{\alpha}_1\tilde{p}_{w6} \end{bmatrix}$$

and

$$C(u_L, u_R)_\Phi = \begin{bmatrix} \bar{w}_3 & 0 & \bar{w}_1 & 0 & 0 & 0 \\ 0 & \bar{w}_4 & 0 & \bar{w}_2 & 0 & 0 \\ \tilde{p}_{w1} & \tilde{p}_{w2} & 2\bar{w}_3 + \tilde{p}_{w3} & 2\bar{w}_4 + \tilde{p}_{w4} & \tilde{p}_{w5} & \tilde{p}_{w6} \\ \bar{\alpha}_1\tilde{p}_{w1} & \bar{\alpha}_1\tilde{p}_{w2} & \bar{\alpha}_1\tilde{p}_{w3} & 2\bar{w}_4 + \bar{\alpha}_1\tilde{p}_{w4} & \bar{\alpha}_1\tilde{p}_{w5} & \bar{\alpha}_1\tilde{p}_{w6} \\ 0 & 0 & \bar{w}_5 & \bar{w}_6 & \bar{w}_3 & \bar{w}_4 \\ 0 & -\bar{w}_4\dfrac{\bar{\rho}}{\rho_1} & 0 & \bar{w}_6 - \bar{w}_2\dfrac{\bar{\rho}}{\rho_1} & 0 & \bar{w}_4 \end{bmatrix}$$

where $\bar{w}_i$ denotes the arithmetic mean of w_i and $\tilde{p}_{wi}$ is an average of the pressure derivative p_{wi} given by

$$\tilde{p}_{wi} = \int_0^1 \frac{\partial p}{\partial w_i}\left(w_L + s\,(w_R - w_L)\right) ds \qquad \text{(III.6)}$$

Finally, using Eq. (II.9), we find the Roe-averaged matrix for the system without virtual mass

$$A_\delta(u) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ \tilde{p}_1 - \tilde{u}_v^2 & \tilde{p}_2 - \tilde{u}_1^2 & \tilde{p}_3 + 2\tilde{u}_v \\ \bar{\alpha}_1\tilde{p}_1 & \bar{\alpha}_1\tilde{p}_2 - \tilde{u}_1^2 & \bar{\alpha}_1\tilde{p}_3 \\ \tilde{u}\tilde{p}_1 - \tilde{u}_v\tilde{H}_v & \tilde{u}\tilde{p}_2 - \tilde{u}_1\tilde{H}_1 + (\tilde{u}_1 - \tilde{u}_v)\,\bar{p}/\rho_1 & \tilde{u}\tilde{p}_3 + \tilde{H}_v \\ \bar{\alpha}_1\tilde{u}_1\tilde{p}_1 & \bar{\alpha}_1\tilde{u}_1\tilde{p}_2 - \tilde{u}_1\tilde{E}_1 & \bar{\alpha}_1\tilde{u}_1\tilde{p}_3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ \tilde{p}_4 + 2(\tilde{u}_1 - \tilde{u}_v) & \tilde{p}_5 & \tilde{p}_6 \\ 2\tilde{u}_1 + \bar{\alpha}_1\tilde{p}_4 & \bar{\alpha}_1\tilde{p}_5 & \bar{\alpha}_1\tilde{p}_6 \\ \tilde{u}\tilde{p}_4 + \tilde{H}_1 - \tilde{H}_v & \tilde{u}\tilde{p}_5 + \tilde{u}_v & \tilde{u}\tilde{p}_6 + \tilde{u}_1 - \tilde{u}_v \\ \bar{\alpha}_1\tilde{u}_1\tilde{p}_4 + \tilde{E}_1 & \bar{\alpha}_1\tilde{u}_1\tilde{p}_5 & \bar{\alpha}_1\tilde{u}_1\tilde{p}_6 + \tilde{u}_1 \end{bmatrix}$$

where $\tilde{p}_i$ is an approximation of the pressure derivative $\partial p/\partial u_i$ given by

$$\tilde{p}_i = \frac{1}{2\bar{w}_i}\tilde{p}_{wi} \qquad \text{(III.7)}$$

and $\tilde{u}_k$, $\tilde{H}_k$ are the Roe-averaged velocity and enthalpy :

$$\tilde{u}_k = \frac{\sqrt{\rho^R c_k^R} u_k^R + \sqrt{\rho^L c_k^L} u_k^L}{\sqrt{\rho^R c_k^R} + \sqrt{\rho^L c_k^L}} \tag{III.8}$$

$$\tilde{H}_k = \frac{\sqrt{\rho^R c_k^R} H_k^R + \sqrt{\rho^L c_k^L} H_k^L}{\sqrt{\rho^R c_k^R} + \sqrt{\rho^L c_k^L}} \tag{III.9}$$

3.2 Two-Fluid Model with Virtual Mass Force Term

We follow a similar method to construct a Roe averaged matrix for the complete system (I.6). We still use the canonical path for the parameter vector w, defined by (III.4), to linearize the matrices $A_\delta(\mathbf{u})$ and $M_\delta(\mathbf{u})$. Then, a routine calculation shows that the Roe-averaged matrix is given as a sum of two averaged matrices

$$A(u_L, u_R)_\Phi^{vm} = A_0(u_L, u_R)_\Phi + \bar{\delta} A_\delta(u_L, u_R)_\Phi \tag{III.10}$$

where the first matrix is the Roe-averaged matrix for the system without virtual mass term and the second is the linearized matrix corresponding to the virtual mass term.

4. Numerical Results

In this section we extend Roe's numerical scheme, for the calculation of one-dimensional two-phase flow based on the two-fluid model. The resulting first-order numerical scheme may be written :

$$\mathbf{u}_i^{t+\Delta t} = \mathbf{u}_i^t + \frac{\Delta t}{\Delta x}\left(F^+(\mathbf{u}_{i-1}^t, \mathbf{u}_i^t) + F^-(\mathbf{u}_i^t, \mathbf{u}_{i+1}^t)\right)$$

with

$$F^\pm(\mathbf{u}_{i-1}^t, \mathbf{u}_i^t) = A_\delta^\pm(\mathbf{u}_{i-1}^t, \mathbf{u}_i^t)_\Phi(\mathbf{u}_i^t - \mathbf{u}_{i-1}^t)$$

The matrices $A_\delta^\pm$ are the positive and the negative part of the Roe-averaged matrix given by

$$A^\pm(\mathbf{u}_{i-1}^t, \mathbf{u}_i^t) = R_{i-1/2}\left(\frac{\Lambda_{i-1/2} \mp |\Lambda_{i-1/2}|}{2}\right) R_{i-1/2}^{-1}$$

where $\Lambda_{i-1/2}$ and $R_{i-1/2}$ are the matrices containing, respectively, the eigenvalues and eigenvectors of the Roe-averaged matrix.

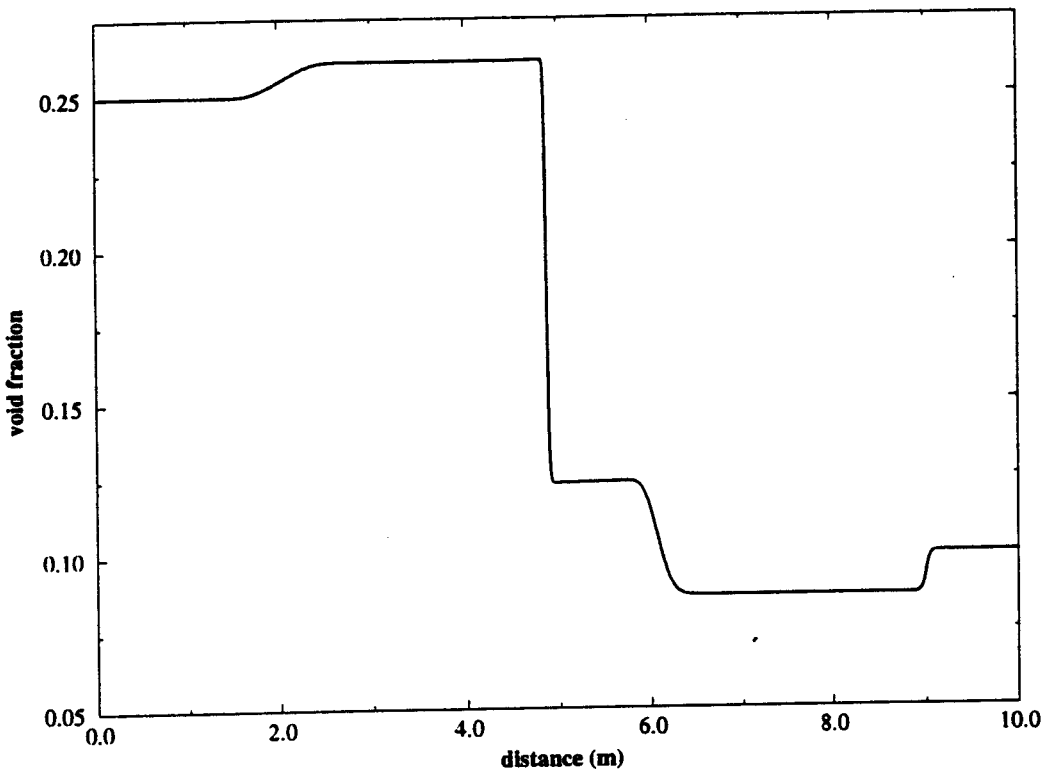
Problem 1 : Shock-tube problem

This problem consists in a Riemann problem for the two-fluid model where the left and right states are given by

State u_L : $p_L = 25\text{MPa}$ $\alpha_L = 0.25$ $u_L = 0.\text{ms}^{-1}$ $u_{rL} = 0.\text{ms}^{-1}$

State u_R : $p_R = 10\text{MPa}$ $\alpha_R = 0.1$ $u_R = 0.\text{ms}^{-1}$ $u_{rR} = 0.\text{ms}^{-1}$

The vapor phase is assumed to be an ideal isentropic gas. The computations have been done with 300 nodes and using a virtual mass coefficient equal to 2. and 50. Fig. 1 and Fig.2. give some flow characteristics in each case. The solution is composed by seven constant states separated by rarefaction waves or shock waves. The propagation velocities of the second and third waves being close to each other for small values of the virtual mass coefficient, these waves are not well separated in Figure 1.



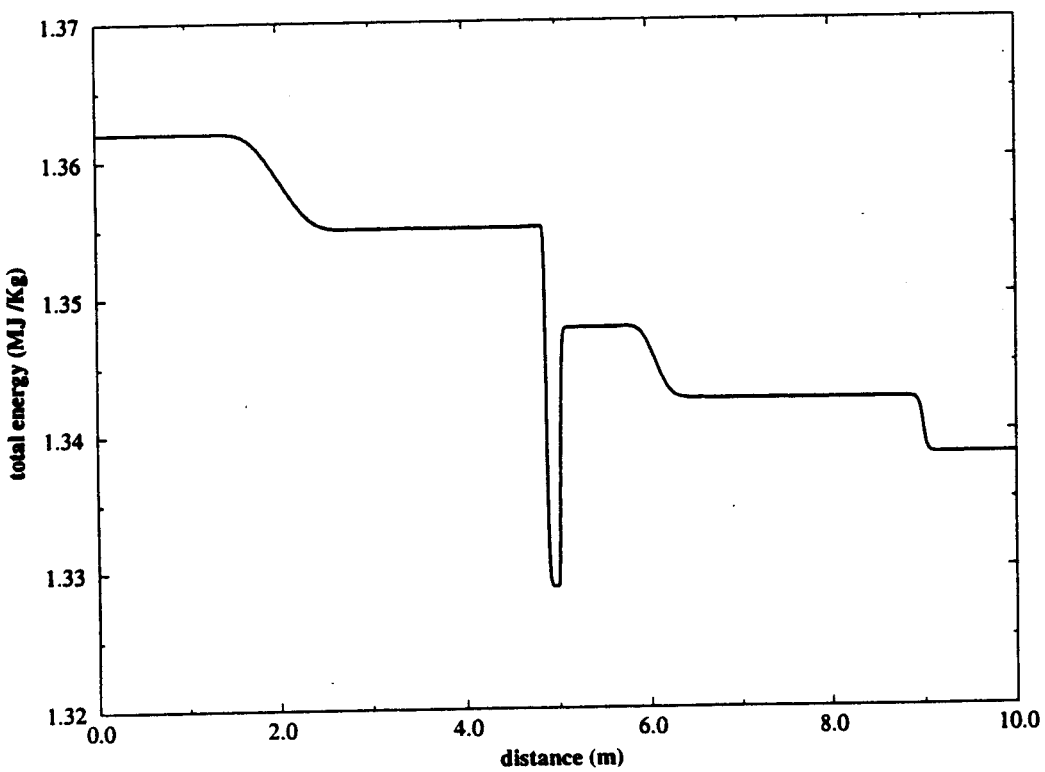
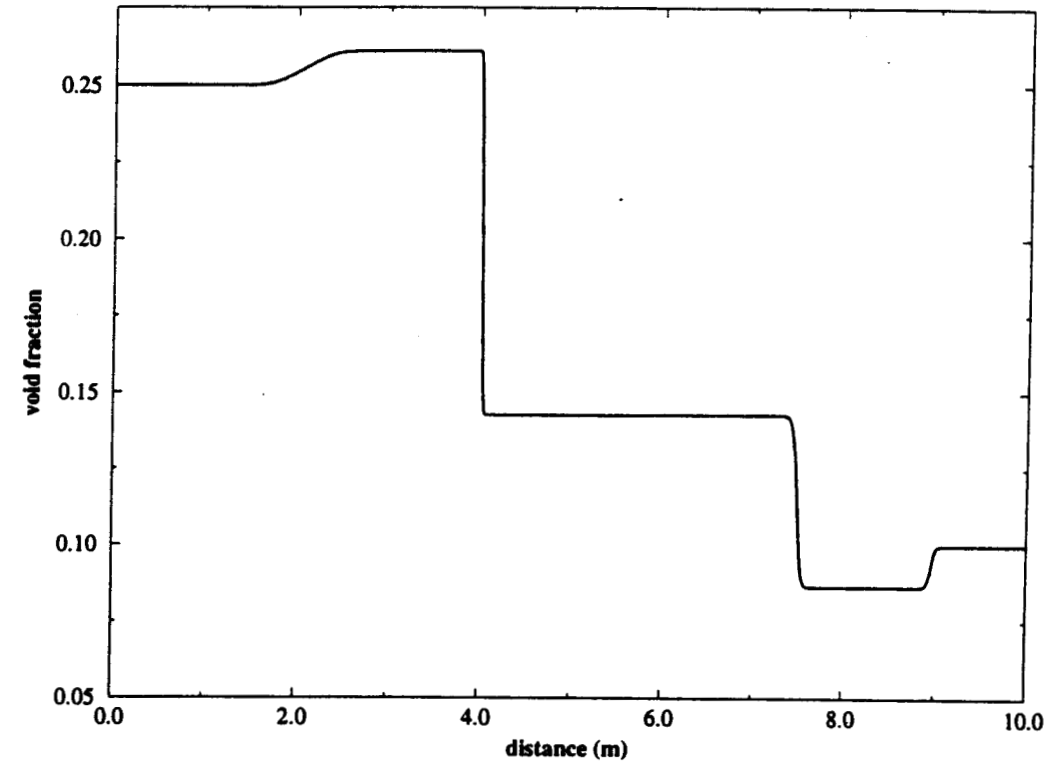


Fig. 1. Shock-tube problem with $c_{vm} = 2$.



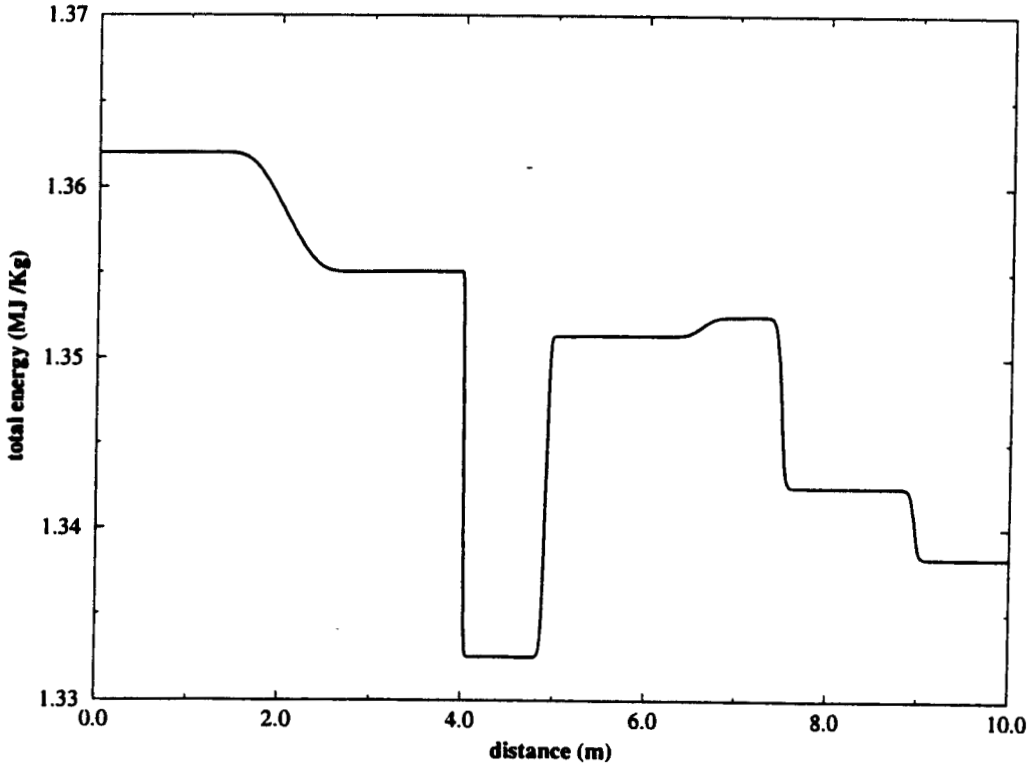


Fig. 2. Shock-tube problem with $c_{vm} = 50$.

Problem 2 : Water faucet problem

This test, proposed by Ransom[10] consists in a vertical water jet, contained within a cylindrical channel, that is accelerated under the action of gravity. At the initial state, the pipe is filled with a uniform column of water surrounded by stagnant vapor, such that the void fraction is 0.2 and the column has a uniform velocity of 10m/s and a uniform pressure of 10^5 Pa.

The boundary conditions are specified velocities of 10m/s for the liquid and 0m/s for the vapor at the inlet, and constant pressure at the outlet. The water faucet problem can be solved analytically by making some further idealizations. This analytical solution was used as a code test in Reference 10.

As illustrated on Figure 3. , a void wave develops and is propagated at liquid velocity. Once the void wave exits the pipe, a steady void profile is established. The calculation was carried out until a steady-state is reached, with 100 nodes and a constant CFL number equals to 0.9. Figure 4. shows the vapor void fraction profile at various time. These results clearly demonstrate the ability of the numerical scheme to capture discontinuities.

In order to test the convergence and the stability character of the scheme, computations have been made using a discretization with 50 and 200 nodes, but constant CFL numbers equal to 0.9. The Figure 5 gives the void fraction profile for the various discretization. An interesting feature of the results shown in Figure 5 is that there is no oscillations at the discontinuity of the void fraction.

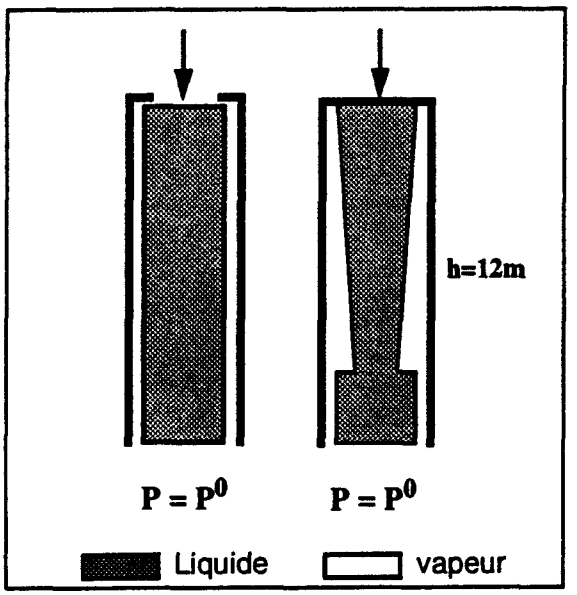


Fig. 3. Schematic of the water faucet problem

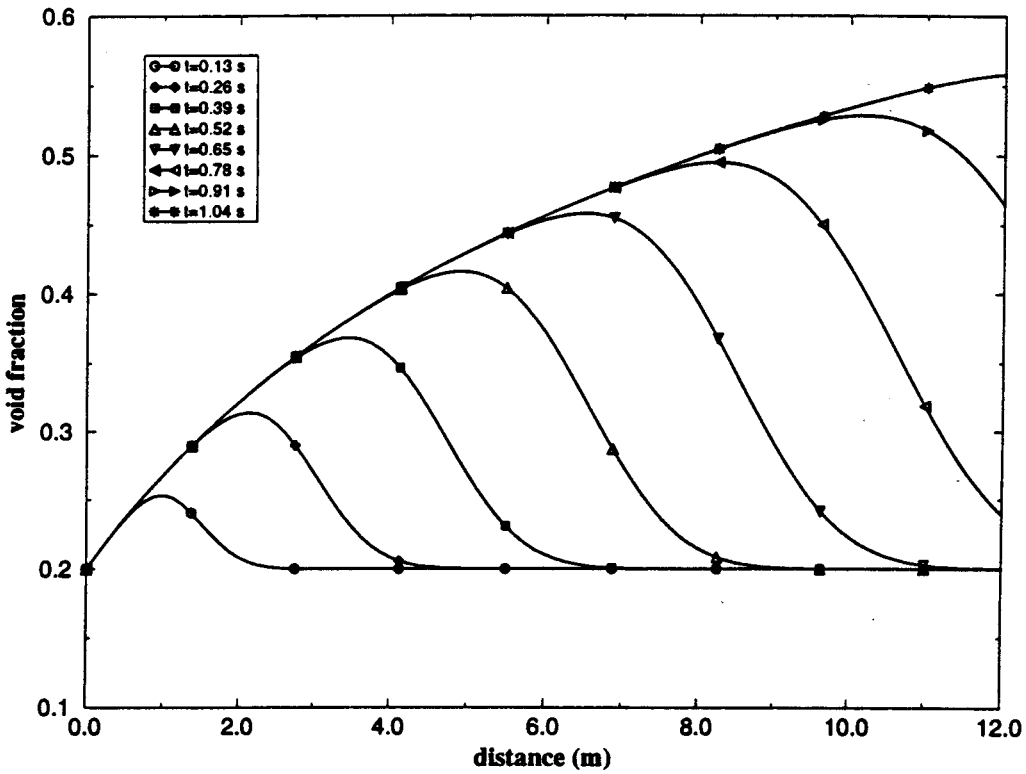


Fig. 4. Void fraction profile for the water fawcet problem

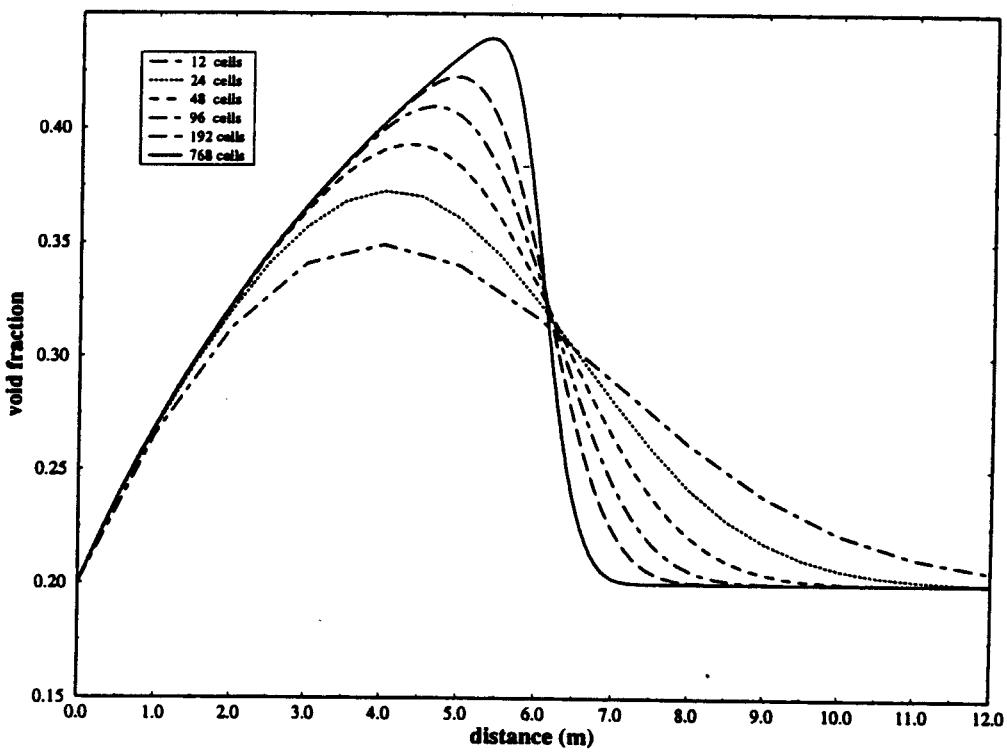


Fig. 5. Void fraction profile for the water fawcet problem (*continued*)

5. References

1. I. Toumi : "An Implicit Upwind Method for 3D Two-Phase Flows Computations of P.W.R. Cores with Irregular Meshes." Proceedings of the ANS International Topical Meeting on Advances in Mathematics, Computations, and Reactor Physics, Pittsburgh, Pennsylvania (1991).

2. V. H. Ransom et al., "RELAP5/MOD1 Code manual Volum 1 : code structure, system models and numerical method", NUREG/CR-1826, EGG-2076 (1982)

3. D. Drew et al., "The analysis of virtual mass effects in two-phase flow", Int. J. Multiphase Flow 5, 233-242 (1979).

4. R. T. Lahey, "The effect of virtual mass on the numerical stability of accelerating two-phase flows", Int. J. Multiphase Flow 6, 281-294 (1980).

5. A. Kumbaro, I. Toumi "An approximate Riemann Solver for a Nonconservative Hyperbolic Two-Fluid Model", To appear.

6. P. L. Roe : "Approximate Riemann solvers, parameter vectors and difference schemes", J. Comput. Phys. 43 357-372 (1981).

7. I. Toumi : "A weak formulation of Roe's approximate Rieman solver", J. Comput. Phys. 102 360-373 (1992).

8. G. Dal Maso, P. Le Floch, F. Murat : "Definition and weak stability of a non-conservative product", Rapport interne, CMAP Ecole Polytechnique, FRANCE (1989).

9. P. Le Floch : "Entropy weak solutions to non-linear hyperbolic systems under non-conservative form", Commun. in part. diff. equ. 13(6) 669-727 (1988).

10. V. H. Ransom, V. Mousseau : "Convergence and accuracy of the RELAP5 two-phase flow model", Proceedings of the ANS International Topical Meeting on Advances in Mathematics, Computations, and Reactor Physics, Pittsburgh, Pennsylvania (1991).